The impact of class imbalance in classification performance metrics based on the binary confusion matrix.

Supplementary material

1. Derivation of $\mu = \mu(\lambda_{PP}, \lambda_{NN}, \pi_P, \pi_N)$.

1. Sensitivity (SNS).

$$SNS = TPR \equiv \frac{TP}{TP + FN} = \frac{m_{PP}}{m_P} = \frac{\lambda_{PP} m_P}{m_P} = \lambda_{PP}. \quad (1)$$

2. Specificity (SPC).

$$SPC = TNR \equiv \frac{TN}{TN + FP} = \frac{m_{NN}}{m_N} = \frac{\lambda_{NN} m_N}{m_N} = \lambda_{NN}. \quad (2)$$

3. Precision (PRC).

$$PRC = PPV \equiv \frac{TP}{TP + FP} = \frac{m_{PP}}{e_P} = \frac{\lambda_{PP} m_P}{\lambda_{PP} m_P + \lambda_{NP} m_N} = \frac{\lambda_{PP}}{\lambda_{PP} + \lambda_{NP} \frac{m_N}{m_P}}. \quad (3)$$

$$PRC = \frac{\lambda_{PP}}{\lambda_{PP} + \lambda_{NP} \frac{\pi_N}{\pi_P}} = \frac{\lambda_{PP} \pi_P}{\lambda_{PP} \pi_P + \lambda_{NP} \pi_N} = \frac{\lambda_{PP} \pi_P}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N}. \quad (4)$$

4. Negative Predictive Value (NPV).

$$NPV \equiv \frac{TN}{TN + FN} = \frac{m_{NN}}{e_N} = \frac{\lambda_{NN} m_N}{\lambda_{NN} m_N + \lambda_{PN} m_P} = \frac{\lambda_{NN}}{\lambda_{NN} + \lambda_{PN} \frac{m_P}{m_N}}. \quad (5)$$

$$NPV = \frac{\lambda_{NN}}{\lambda_{NN} + \lambda_{PN} \frac{\pi_P}{\pi_N}} = \frac{\lambda_{NN} \pi_N}{\lambda_{NN} \pi_N + (1 - \lambda_{PP}) \pi_P}. \quad (6)$$

5. Accuracy (ACC).

$$ACC \equiv \frac{TP + TN}{TP + FN + TN + FP} = \frac{m_{PP} + m_{NN}}{m}. \quad (7)$$

$$ACC = \frac{\lambda_{PP} m_P + \lambda_{NN} m_N}{m} = \lambda_{PP} \pi_P + \lambda_{NN} \pi_N. \quad (8)$$

6. F_1 score.

$$F_1 \equiv 2 \frac{PRC \cdot SNS}{PRC + SNS} = 2 \frac{\frac{\lambda_{PP} \pi_P}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N} \cdot \lambda_{PP}}{\frac{\lambda_{PP} \pi_P}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N} + \lambda_{PP}} = 2 \frac{\frac{\lambda_{PP} \pi_P}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N}}{\frac{\pi_P}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N} + 1}. \quad (9)$$

$$F_1 = 2 \frac{\frac{\lambda_{PP} \pi_P}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N}}{\frac{\pi_P + \lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N}{\lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N}} = 2 \frac{\lambda_{PP} \pi_P}{\pi_P + \lambda_{PP} \pi_P + (1 - \lambda_{NN}) \pi_N}. \quad (10)$$

$$F_1 = 2 \frac{\lambda_{PP} \pi_P}{(1 + \lambda_{PP}) \pi_P + (1 - \lambda_{NN}) \pi_N}. \quad (11)$$

7. Geometric Mean (GM).

$$GM \equiv \sqrt{SNS \cdot SPC} = \sqrt{\lambda_{PP} \cdot \lambda_{NN}}. \tag{12}$$

8. Matthews Correlation Coefficient (MCC).

$$MCC = \frac{TP \cdot TN - FP \cdot FN}{\sqrt{(TP + FP)(TP + FN)(TN + FP)(TN + FN)}}. \tag{13}$$

$$MCC = \frac{\lambda_{PP}m_P \cdot \lambda_{NN}m_N - \lambda_{NP}m_N \cdot \lambda_{PN}m_P}{\sqrt{(\lambda_{PP}m_P + \lambda_{NP}m_N)(\lambda_{PP}m_P + \lambda_{PN}m_P)(\lambda_{NN}m_N + \lambda_{NP}m_N)(\lambda_{NN}m_N + \lambda_{PN}m_P)}}. \tag{14}$$

$$MCC = \frac{m_P m_N(\lambda_{PP}\lambda_{NN} - \lambda_{NP}\lambda_{PN})}{\sqrt{(\lambda_{PP}m_P + \lambda_{NP}m_N)(\lambda_{PP}m_P + \lambda_{PN}m_P)(\lambda_{NN}m_N + \lambda_{NP}m_N)(\lambda_{NN}m_N + \lambda_{PN}m_P)}}. \tag{15}$$

$$MCC = \frac{\lambda_{PP}\lambda_{NN} - \lambda_{NP}\lambda_{PN}}{\sqrt{\frac{(\lambda_{PP}m_P + \lambda_{NP}m_N)(\lambda_{PP}m_P + \lambda_{PN}m_P)(\lambda_{NN}m_N + \lambda_{NP}m_N)(\lambda_{NN}m_N + \lambda_{PN}m_P)}{m_P m_N \cdot m_P m_N}}}. \tag{16}$$

$$MCC = \frac{\lambda_{PP}\lambda_{NN} - \lambda_{NP}\lambda_{PN}}{\sqrt{\frac{(\lambda_{PP}m_P + \lambda_{PN}m_P)}{m_P} \cdot \frac{(\lambda_{NN}m_N + \lambda_{NP}m_N)}{m_N} \cdot \frac{(\lambda_{PP}m_P + \lambda_{NP}m_N)}{m_P} \cdot \frac{(\lambda_{NN}m_N + \lambda_{PN}m_P)}{m_N}}}. \tag{17}$$

$$MCC = \frac{\lambda_{PP}\lambda_{NN} - \lambda_{NP}\lambda_{PN}}{\sqrt{(\lambda_{PP} + \lambda_{PN})(\lambda_{NN} + \lambda_{NP})\left(\lambda_{PP} + \lambda_{NP}\frac{m_N}{m_P}\right)\left(\lambda_{NN} + \lambda_{PN}\frac{m_P}{m_N}\right)}}. \tag{18}$$

$$MCC = \frac{\lambda_{PP}\lambda_{NN} - (1 - \lambda_{NN})(1 - \lambda_{PP})}{\sqrt{1 \cdot 1 \cdot \left[\lambda_{PP} + (1 - \lambda_{NN})\frac{\pi_N}{\pi_P}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{\pi_P}{\pi_N}\right]}}. \tag{19}$$

$$MCC = \frac{\lambda_{PP}\lambda_{NN} - (1 - \lambda_{PP} - \lambda_{NN} + \lambda_{PP}\lambda_{NN})}{\sqrt{\left[\lambda_{PP} + (1 - \lambda_{NN})\frac{\pi_N}{\pi_P}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{\pi_P}{\pi_N}\right]}}. \tag{20}$$

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{\left[\lambda_{PP} + (1 - \lambda_{NN})\frac{\pi_N}{\pi_P}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{\pi_P}{\pi_N}\right]}}. \tag{21}$$

9. Bookmaker Informedness (BM).

$$BM \equiv SNS + SPC - 1 = \lambda_{PP} + \lambda_{NN} - 1. \tag{22}$$

10. Markedness (MK).

$$MK \equiv PPV + NPV - 1 = \frac{\lambda_{PP}\pi_P}{\lambda_{PP}\pi_P + (1 - \lambda_{NN})\pi_N} + \frac{\lambda_{NN}\pi_N}{\lambda_{NN}\pi_N + (1 - \lambda_{PP})\pi_P} - 1. \tag{23}$$

$$MK = \frac{\pi_P}{\pi_P + \frac{(1 - \lambda_{NN})}{\lambda_{PP}}\pi_N} + \frac{\pi_N}{\pi_N + \frac{(1 - \lambda_{PP})}{\lambda_{NN}}\pi_P} - 1. \tag{24}$$

2. Derivation of $\mu = \mu(\lambda_{PP}, \lambda_{NN})$ when the classes are balanced.

1. Sensitivity (SNS).

$$SNS = \lambda_{PP}. \tag{25}$$

2. Specificity (SPC).

$$SPC = \lambda_{NN}. \tag{26}$$

3. Precision (PRC).

$$PRC = \frac{\lambda_{PP}\pi_P}{\lambda_{PP}\pi_P + (1 - \lambda_{NN})\pi_N} = \frac{\lambda_{PP} \cdot \frac{1}{2}}{\lambda_{PP} \cdot \frac{1}{2} + (1 - \lambda_{NN}) \cdot \frac{1}{2}} = \frac{\lambda_{PP}}{\lambda_{PP} + (1 - \lambda_{NN})}. \tag{27}$$

4. Negative Predictive Value (NPV).

$$NPV = \frac{\lambda_{NN}\pi_N}{\lambda_{NN}\pi_N + (1 - \lambda_{PP})\pi_P} = \frac{\lambda_{NN} \cdot \frac{1}{2}}{\lambda_{NN} \cdot \frac{1}{2} + (1 - \lambda_{PP}) \cdot \frac{1}{2}} = \frac{\lambda_{NN}}{\lambda_{NN} + (1 - \lambda_{PP})}. \tag{28}$$

5. Accuracy (ACC).

$$ACC = \lambda_{PP}\pi_P + \lambda_{NN}\pi_N = \lambda_{PP} \cdot \frac{1}{2} + \lambda_{NN} \cdot \frac{1}{2} = \frac{1}{2}(\lambda_{PP} + \lambda_{NN}). \tag{29}$$

6. F_1 score.

$$F_1 = 2\frac{\lambda_{PP} \cdot \frac{1}{2}}{(1 + \lambda_{PP}) \cdot \frac{1}{2} + (1 - \lambda_{NN}) \cdot \frac{1}{2}} = \frac{2\lambda_{PP}}{(1 + \lambda_{PP}) + (1 - \lambda_{NN})}. \tag{30}$$

7. Geometric Mean (GM).

$$GM = \sqrt{\lambda_{PP} \cdot \lambda_{NN}}. \tag{31}$$

8. Matthews Correlation Coefficient (MCC).

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{\left[\lambda_{PP} + (1 - \lambda_{NN})\frac{\pi_N}{\pi_P}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{\pi_P}{\pi_N}\right]}}. \tag{32}$$

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{\left[\lambda_{PP} + (1 - \lambda_{NN})\frac{0.5}{0.5}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{0.5}{0.5}\right]}}. \tag{33}$$

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{[\lambda_{PP} + (1 - \lambda_{NN})][\lambda_{NN} + (1 - \lambda_{PP})]}}. \tag{34}$$

9. Bookmaker Informedness (BM).

$$BM = \lambda_{PP} + \lambda_{NN} - 1. \tag{35}$$

10. Markedness (MK).

$$MK = \frac{\pi_P}{\pi_P + \frac{(1 - \lambda_{NN})}{\lambda_{PP}}\pi_N} + \frac{\pi_N}{\pi_N + \frac{(1 - \lambda_{PP})}{\lambda_{NN}}\pi_P} - 1. \tag{36}$$

$$MK = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{(1-\lambda_{NN})}{\lambda_{PP}} \cdot \frac{1}{2}} + \frac{\frac{1}{2}}{\frac{1}{2} + \frac{(1-\lambda_{PP})}{\lambda_{NN}} \cdot \frac{1}{2}} - 1. \tag{37}$$

$$MK = \frac{\frac{1}{2}1}{1 + \frac{(1-\lambda_{NN})}{\lambda_{PP}}} + \frac{1}{1 + \frac{(1-\lambda_{PP})}{\lambda_{NN}}} - 1. \tag{38}$$

3. Derivation of $\mu = \mu(\lambda_{PP}, \lambda_{NN}, \delta)$.

1. Sensitivity (SNS).

$$SNS = \lambda_{PP}. \tag{39}$$

2. Specificity (SPC).

$$SPC = \lambda_{NN}. \tag{40}$$

3. Precision (PRC).

$$PRC = \frac{\lambda_{PP}\pi_P}{\lambda_{PP}\pi_P + (1-\lambda_{NN})\pi_N} = \frac{\lambda_{PP}\frac{1+\delta}{2}}{\lambda_{PP}\frac{1+\delta}{2} + (1-\lambda_{NN})\frac{1-\delta}{2}}. \tag{41}$$

$$PRC = \frac{\lambda_{PP}(1+\delta)}{\lambda_{PP}(1+\delta) + (1-\lambda_{NN})(1-\delta)}. \tag{42}$$

4. Negative Predictive Value (NPV).

$$NPV = \frac{\lambda_{NN}\pi_N}{\lambda_{NN}\pi_N + (1-\lambda_{PP})\pi_P} = \frac{\lambda_{NN}\frac{1-\delta}{2}}{\lambda_{NN}\frac{1-\delta}{2} + (1-\lambda_{PP})\frac{1+\delta}{2}}. \tag{43}$$

$$NPV = \frac{\lambda_{NN}(1-\delta)}{\lambda_{NN}(1-\delta) + (1-\lambda_{PP})(1+\delta)}. \tag{44}$$

5. Accuracy (ACC).

$$ACC = \lambda_{PP}\frac{1+\delta}{2} + \lambda_{NN}\frac{1-\delta}{2} = \frac{1}{2}[\lambda_{PP}(1+\delta) + \lambda_{NN}(1-\delta)]. \tag{45}$$

6. F_1 score.

$$F_1 = 2\frac{\lambda_{PP}\frac{1+\delta}{2}}{(1+\lambda_{PP})\frac{1+\delta}{2} + (1-\lambda_{NN})\frac{1-\delta}{2}} = \frac{2\,\lambda_{PP}(1+\delta)}{(1+\lambda_{PP})(1+\delta) + (1-\lambda_{NN})(1-\delta)}. \tag{46}$$

7. Geometric Mean (GM).

$$GM \equiv \sqrt{SNS \cdot SPC} = \sqrt{\lambda_{PP} \cdot \lambda_{NN}}. \tag{47}$$

8. Matthews Correlation Coefficient (MCC).

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{\left[\lambda_{PP} + (1-\lambda_{NN})\frac{\pi_N}{\pi_P}\right]\left[\lambda_{NN} + (1-\lambda_{PP})\frac{\pi_P}{\pi_N}\right]}}. \tag{48}$$

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{\left[\lambda_{PP} + (1 - \lambda_{NN})\frac{\frac{1-\delta}{2}}{\frac{1+\delta}{2}}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{\frac{1+\delta}{2}}{\frac{1-\delta}{2}}\right]}}. \tag{49}$$

$$MCC = \frac{\lambda_{PP} + \lambda_{NN} - 1}{\sqrt{\left[\lambda_{PP} + (1 - \lambda_{NN})\frac{1-\delta}{1+\delta}\right]\left[\lambda_{NN} + (1 - \lambda_{PP})\frac{1+\delta}{1-\delta}\right]}}. \tag{50}$$

9. Bookmaker Informedness (BM).

$$BM = \lambda_{PP} + \lambda_{NN} - 1. \tag{51}$$

10. Markedness (MK).

$$MK = MK = \frac{\pi_P}{\pi_P + \frac{(1-\lambda_{NN})}{\lambda_{PP}}\pi_N} + \frac{\pi_N}{\pi_N + \frac{(1-\lambda_{PP})}{\lambda_{NN}}\pi_P} - 1. \tag{52}$$

$$MK = MK = \frac{\frac{1+\delta}{2}}{\frac{1+\delta}{2} + \frac{(1-\lambda_{NN})}{\lambda_{PP}}\frac{1-\delta}{2}} + \frac{\frac{1-\delta}{2}}{\frac{1-\delta}{2} + \frac{(1-\lambda_{PP})}{\lambda_{NN}}\frac{1+\delta}{2}} - 1. \tag{53}$$

$$MK = MK = \frac{1+\delta}{(1+\delta) + \frac{(1-\lambda_{NN})}{\lambda_{PP}}(1-\delta)} + \frac{1-\delta}{(1-\delta) + \frac{(1-\lambda_{PP})}{\lambda_{NN}}(1+\delta)} - 1. \tag{54}$$

4. Derivation of $B_\mu = B_\mu(\lambda_{PP}, \lambda_{NN}, \delta)$.

1. Sensitivity (SNS).

$$B_{SNS}(\lambda_{PP}, \lambda_{NN}, \delta) = SNS(\lambda_{PP}, \lambda_{NN}, \delta) - SNS_b(\lambda_{PP}, \lambda_{NN}, \delta) = \lambda_{PP} - \lambda_{PP} = 0. \tag{55}$$

2. Specificity (SPC).

$$B_{SPC}(\lambda_{PP}, \lambda_{NN}, \delta) = SPC(\lambda_{PP}, \lambda_{NN}, \delta) - SPC_b(\lambda_{PP}, \lambda_{NN}, \delta) = \lambda_{NN} - \lambda_{NN} = 0. \tag{56}$$

3. Precision (PRC).

$$B_{PRC}(\lambda_{PP}, \lambda_{NN}, \delta) = PRC(\lambda_{PP}, \lambda_{NN}, \delta) - PRC_b(\lambda_{PP}, \lambda_{NN}, \delta). \tag{57}$$

$$B_{PRC}(\lambda_{PP}, \lambda_{NN}, \delta) = \frac{\lambda_{PP}(1+\delta)}{\lambda_{PP}(1+\delta) + (1-\lambda_{NN})(1-\delta)} - \frac{\lambda_{PP}}{\lambda_{PP} + (1-\lambda_{NN})}. \tag{58}$$

$$B_{PRC}(\lambda_{PP}, \lambda_{NN}, \delta) = \frac{1+\delta}{(1+\delta) + \frac{1-\lambda_{NN}}{\lambda_{PP}}(1-\delta)} - \frac{1}{1 + \frac{1-\lambda_{NN}}{\lambda_{PP}}}. \tag{59}$$

4. Negative Predictive Value (NPV).

$$B_{NPV}(\lambda_{PP}, \lambda_{NN}, \delta) = NPV(\lambda_{PP}, \lambda_{NN}, \delta) - NPV_b(\lambda_{PP}, \lambda_{NN}, \delta). \tag{60}$$

$$B_{NPV}(\lambda_{PP}, \lambda_{NN}, \delta) = \frac{\lambda_{NN}(1-\delta)}{\lambda_{NN}(1-\delta) + (1-\lambda_{PP})(1+\delta)} - \frac{\lambda_{NN}}{\lambda_{NN} + (1-\lambda_{PP})}. \tag{61}$$

$$B_{NPV}(\lambda_{PP},\lambda_{NN},\delta) = \frac{1-\delta}{(1-\delta)+\frac{1-\lambda_{PP}}{\lambda_{NN}}(1+\delta)} - \frac{1}{1+\frac{1-\lambda_{PP}}{\lambda_{NN}}}. \quad (62)$$

5. Accuracy (ACC).

$$B_{ACC}(\lambda_{PP},\lambda_{NN},\delta) = ACC(\lambda_{PP},\lambda_{NN},\delta) - ACC_b(\lambda_{PP},\lambda_{NN},\delta). \quad (63)$$

$$B_{ACC}(\lambda_{PP},\lambda_{NN},\delta) = \lambda_{PP}\frac{1+\delta}{2} + \lambda_{NN}\frac{1-\delta}{2} - \frac{\lambda_{PP}+\lambda_{NN}}{2}. \quad (64)$$

$$B_{ACC}(\lambda_{PP},\lambda_{NN},\delta) = \frac{\lambda_{PP}}{2} + \frac{\lambda_{PP}\delta}{2} + \frac{\lambda_{NN}}{2} - \frac{\lambda_{NN}\delta}{2} - \frac{\lambda_{PP}}{2} - \frac{\lambda_{NN}}{2} = \frac{\lambda_{PP}\delta}{2} - \frac{\lambda_{NN}\delta}{2}. \quad (65)$$

$$B_{ACC}(\lambda_{PP},\lambda_{NN},\delta) = \frac{\delta}{2}(\lambda_{PP}-\lambda_{NN}). \quad (66)$$

6. F_1 score.

$$B_{F1}(\lambda_{PP},\lambda_{NN},\delta) = F_1(\lambda_{PP},\lambda_{NN},\delta) - F_{1b}(\lambda_{PP},\lambda_{NN},\delta). \quad (67)$$

$$B_{F1}(\lambda_{PP},\lambda_{NN},\delta) = \frac{2\lambda_{PP}(1+\delta)}{(1+\lambda_{PP})(1+\delta)+(1-\lambda_{NN})(1-\delta)} - \frac{2\lambda_{PP}}{2+\lambda_{PP}-\lambda_{NN}}. \quad (68)$$

7. Geometric Mean (GM).

$$B_{GM}(\lambda_{PP},\lambda_{NN},\delta) = GM(\lambda_{PP},\lambda_{NN},\delta) - GM_b(\lambda_{PP},\lambda_{NN},\delta) = \sqrt{\lambda_{PP}\cdot\lambda_{NN}} - \sqrt{\lambda_{PP}\cdot\lambda_{NN}} = 0. \quad (69)$$

8. Matthews Correlation Coefficient (MCC).

$$B_{MCC}(\lambda_{PP},\lambda_{NN},\delta) = MCC(\lambda_{PP},\lambda_{NN},\delta) - MCC_b(\lambda_{PP},\lambda_{NN},\delta). \quad (70)$$

$$B_{MCC}(\lambda_{PP},\lambda_{NN},\delta) = \frac{\lambda_{PP}+\lambda_{NN}-1}{\sqrt{\left[\lambda_{PP}+(1-\lambda_{NN})\frac{1-\delta}{1+\delta}\right]\left[\lambda_{NN}+(1-\lambda_{PP})\frac{1+\delta}{1-\delta}\right]}} - \frac{\lambda_{PP}+\lambda_{NN}-1}{\sqrt{[\lambda_{PP}+(1-\lambda_{NN})][\lambda_{NN}+(1-\lambda_{PP})]}}. \quad (71)$$

$$B_{MCCn}(\lambda_{PP},\lambda_{NN},\delta) = MCCn(\lambda_{PP},\lambda_{NN},\delta) - MCCn_b(\lambda_{PP},\lambda_{NN}). \quad (72)$$

$$B_{MCCn}(\lambda_{PP},\lambda_{NN},\delta) = \frac{MCC(\lambda_{PP},\lambda_{NN},\delta)+1}{2} - \frac{MCC_b(\lambda_{PP},\lambda_{NN})+1}{2}. \quad (73)$$

$$B_{MCCn}(\lambda_{PP},\lambda_{NN},\delta) = \frac{MCC(\lambda_{PP},\lambda_{NN},\delta) - MCC_b(\lambda_{PP},\lambda_{NN})}{2}. \quad (74)$$

$$B_{MCCn}(\lambda_{PP},\lambda_{NN},\delta) = \frac{B_{MCC}(\lambda_{PP},\lambda_{NN},\delta)}{2}. \quad (75)$$

$$B_{MCCn}(\lambda_{PP},\lambda_{NN},\delta) = \frac{\lambda_{PP}+\lambda_{NN}-1}{2\sqrt{\left[\lambda_{PP}+(1-\lambda_{NN})\frac{1-\delta}{1+\delta}\right]\left[\lambda_{NN}+(1-\lambda_{PP})\frac{1+\delta}{1-\delta}\right]}} - \frac{\lambda_{PP}+\lambda_{NN}-1}{2\sqrt{[\lambda_{PP}+(1-\lambda_{NN})][\lambda_{NN}+(1-\lambda_{PP})]}}. \quad (76)$$

9. Bookmaker Informedness (BM).

$$B_{BM}(\lambda_{PP}, \lambda_{NN}, \delta) = BM(\lambda_{PP}, \lambda_{NN}, \delta) - BM_b(\lambda_{PP}, \lambda_{NN}, \delta). \tag{77}$$

$$B_{BM}(\lambda_{PP}, \lambda_{NN}, \delta) = (\lambda_{PP} + \lambda_{NN} - 1) - (\lambda_{PP} + \lambda_{NN} - 1) = 0. \tag{78}$$

10. Markedness (MK).

$$B_{MK}(\lambda_{PP}, \lambda_{NN}, \delta) = MK(\lambda_{PP}, \lambda_{NN}, \delta) - MK_b(\lambda_{PP}, \lambda_{NN}, \delta). \tag{79}$$

$$\begin{aligned} B_{MK}(\lambda_{PP}, \lambda_{NN}, \delta) = {} & [PRC(\lambda_{PP}, \lambda_{NN}, \delta) + NPV(\lambda_{PP}, \lambda_{NN}, \delta) - 1] \\ & - [PRC_b(\lambda_{PP}, \lambda_{NN}) + NPV_b(\lambda_{PP}, \lambda_{NN}) - 1]. \end{aligned} \tag{80}$$

$$\begin{aligned} B_{MK}(\lambda_{PP}, \lambda_{NN}, \delta) = {} & [PRC(\lambda_{PP}, \lambda_{NN}, \delta) - PRC_b(\lambda_{PP}, \lambda_{NN})] \\ & + [NPV(\lambda_{PP}, \lambda_{NN}, \delta) - NPV_b(\lambda_{PP}, \lambda_{NN})]. \end{aligned} \tag{81}$$

$$B_{MK}(\lambda_{PP}, \lambda_{NN}, \delta) = B_{PRC}(\lambda_{PP}, \lambda_{NN}, \delta) + B_{NPV}(\lambda_{PP}, \lambda_{NN}, \delta). \tag{82}$$

$$\begin{aligned} B_{MK}(\lambda_{PP}, \lambda_{NN}, \delta) = {} & \frac{1+\delta}{(1+\delta) + \frac{1-\lambda_{NN}}{\lambda_{PP}}(1-\delta)} - \frac{1}{1 + \frac{1-\lambda_{NN}}{\lambda_{PP}}} + \frac{1-\delta}{(1-\delta) + \frac{1-\lambda_{PP}}{\lambda_{NN}}(1+\delta)} \\ & - \frac{1}{1 + \frac{1-\lambda_{PP}}{\lambda_{NN}}}. \end{aligned} \tag{83}$$

$$B_{MKn}(\lambda_{PP}, \lambda_{NN}, \delta) = MKn(\lambda_{PP}, \lambda_{NN}, \delta) - MKn_b(\lambda_{PP}, \lambda_{NN}). \tag{84}$$

$$B_{MKn}(\lambda_{PP}, \lambda_{NN}, \delta) = \frac{MK(\lambda_{PP}, \lambda_{NN}, \delta) + 1}{2} - \frac{MK_b(\lambda_{PP}, \lambda_{NN}) + 1}{2}. \tag{85}$$

$$B_{MKn}(\lambda_{PP}, \lambda_{NN}, \delta) = \frac{MK(\lambda_{PP}, \lambda_{NN}, \delta) - MK_b(\lambda_{PP}, \lambda_{NN})}{2} = \frac{B_{MK}(\lambda_{PP}, \lambda_{NN}, \delta)}{2}. \tag{86}$$

$$B_{MKn}(\lambda_{PP}, \lambda_{NN}, \delta) = \frac{B_{PRC}(\lambda_{PP}, \lambda_{NN}, \delta) + B_{NPV}(\lambda_{PP}, \lambda_{NN}, \delta)}{2}. \tag{87}$$

$$\begin{aligned} & B_{MKn}(\lambda_{PP}, \lambda_{NN}, \delta) \\ & = \frac{1}{2}\left(\frac{1+\delta}{(1+\delta) + \frac{1-\lambda_{NN}}{\lambda_{PP}}(1-\delta)} - \frac{1}{1 + \frac{1-\lambda_{NN}}{\lambda_{PP}}} + \frac{1-\delta}{(1-\delta) + \frac{1-\lambda_{PP}}{\lambda_{NN}}(1+\delta)} \right. \\ & \left. - \frac{1}{1 + \frac{1-\lambda_{PP}}{\lambda_{NN}}} \right). \end{aligned} \tag{88}$$