**The impact of class imbalance in classification performance metrics based on the binary confusion matrix.**

# Supplementary material

## Derivation of $μ=μ\left(λ\_{PP},λ\_{NN},π\_{P},π\_{N}\right)$.

1. Sensitivity ($SNS$).

|  |  |
| --- | --- |
| $$SNS=TPR≡\frac{TP}{TP+FN}=\frac{m\_{PP}}{m\_{P}}=\frac{λ\_{PP}m\_{P}}{m\_{P}}=λ\_{PP}.$$ | (1) |
|  |  |

1. Specificity ($SPC$).

|  |  |
| --- | --- |
| $$SPC=TNR≡\frac{TN}{TN+FP}=\frac{m\_{NN}}{m\_{N}}=\frac{λ\_{NN}m\_{N}}{m\_{N}}=λ\_{NN}.$$ | (2) |
|  |  |

1. Precision ($PRC$).

|  |  |
| --- | --- |
| $$PRC=PPV≡\frac{TP}{TP+FP}=\frac{m\_{PP}}{e\_{P}}=\frac{λ\_{PP}m\_{P}}{λ\_{PP}m\_{P}+λ\_{NP}m\_{N}}=\frac{λ\_{PP}}{λ\_{PP}+λ\_{NP}\frac{m\_{N}}{m\_{P}}}.$$ | (3) |

|  |  |
| --- | --- |
| $$PRC=\frac{λ\_{PP}}{λ\_{PP}+λ\_{NP}\frac{π\_{N}}{π\_{P}}}=\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+λ\_{NP}π\_{N}}=\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}.$$ | (4) |
|  |  |

1. Negative Predictive Value ($NPV$).

|  |  |
| --- | --- |
| $$NPV≡\frac{TN}{TN+FN}=\frac{m\_{NN}}{e\_{N}}=\frac{λ\_{NN}m\_{N}}{λ\_{NN}m\_{N}+λ\_{PN}m\_{P}}=\frac{λ\_{NN}}{λ\_{NN}+λ\_{PN}\frac{m\_{P}}{m\_{N}}}.$$ | (5) |

|  |  |
| --- | --- |
| $$NPV=\frac{λ\_{NN}}{λ\_{NN}+λ\_{PN}\frac{π\_{P}}{π\_{N}}}=\frac{λ\_{NN}π\_{N}}{λ\_{NN}π\_{N}+\left(1-λ\_{PP}\right)π\_{P}}.$$ | (6) |
|  |  |

1. Accuracy ($ACC$).

|  |  |
| --- | --- |
| $$ACC≡\frac{TP+TN}{TP+FN+TN+FP}=\frac{m\_{PP}+m\_{NN}}{m}.$$ | (7) |

|  |  |
| --- | --- |
| $$ACC=\frac{λ\_{PP}m\_{P}+λ\_{NN}m\_{N}}{m}=λ\_{PP}π\_{P}+λ\_{NN}π\_{N}.$$ | (8) |

1. $F\_{1}$ score.

|  |  |
| --- | --- |
| $$F\_{1}≡2\frac{PRC·SNS}{PRC+SNS}=2 \frac{\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}·λ\_{PP}}{\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}+λ\_{PP}}=2 \frac{\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}}{\frac{π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}+1}.$$ | (9) |

|  |  |
| --- | --- |
| $$F\_{1}=2 \frac{\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}}{\frac{π\_{P}+λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}}=2 \frac{λ\_{PP}π\_{P}}{π\_{P}+λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}.$$ | (10) |

|  |  |
| --- | --- |
| $$F\_{1}=2 \frac{λ\_{PP}π\_{P}}{\left(1+λ\_{PP}\right)π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}.$$ | (11) |

1. Geometric Mean ($GM$).

|  |  |
| --- | --- |
| $$GM≡\sqrt{SNS·SPC}=\sqrt{λ\_{PP}·λ\_{NN}}.$$ | (12) |

1. Matthews Correlation Coefficient ($MCC$).

|  |  |
| --- | --- |
| $$MCC=\frac{TP·TN-FP·FN}{\sqrt{\left(TP+FP\right)\left(TP+FN\right)\left(TN+FP\right)\left(TN+FN\right)}}.$$ | (13) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}m\_{P}·λ\_{NN}m\_{N}-λ\_{NP}m\_{N}·λ\_{PN}m\_{P}}{\sqrt{\left(λ\_{PP}m\_{P}+λ\_{NP}m\_{N}\right)\left(λ\_{PP}m\_{P}+λ\_{PN}m\_{P}\right)\left(λ\_{NN}m\_{N}+λ\_{NP}m\_{N}\right)\left(λ\_{NN}m\_{N}+λ\_{PN}m\_{P}\right)}}.$$ | (14) |

|  |  |
| --- | --- |
| $$MCC=\frac{m\_{P}m\_{N}\left(λ\_{PP}λ\_{NN}-λ\_{NP}λ\_{PN}\right)}{\sqrt{\left(λ\_{PP}m\_{P}+λ\_{NP}m\_{N}\right)\left(λ\_{PP}m\_{P}+λ\_{PN}m\_{P}\right)\left(λ\_{NN}m\_{N}+λ\_{NP}m\_{N}\right)\left(λ\_{NN}m\_{N}+λ\_{PN}m\_{P}\right)}}.$$ | (15) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}λ\_{NN}-λ\_{NP}λ\_{PN}}{\sqrt{\frac{\left(λ\_{PP}m\_{P}+λ\_{NP}m\_{N}\right)\left(λ\_{PP}m\_{P}+λ\_{PN}m\_{P}\right)\left(λ\_{NN}m\_{N}+λ\_{NP}m\_{N}\right)\left(λ\_{NN}m\_{N}+λ\_{PN}m\_{P}\right)}{m\_{P}m\_{N}·m\_{P}m\_{N}}}}.$$ | (16) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}λ\_{NN}-λ\_{NP}λ\_{PN}}{\sqrt{\frac{\left(λ\_{PP}m\_{P}+λ\_{PN}m\_{P}\right)}{m\_{P}}·\frac{\left(λ\_{NN}m\_{N}+λ\_{NP}m\_{N}\right)}{m\_{N}}·\frac{\left(λ\_{PP}m\_{P}+λ\_{NP}m\_{N}\right)}{m\_{P}}·\frac{\left(λ\_{NN}m\_{N}+λ\_{PN}m\_{P}\right)}{m\_{N}}}}.$$ | (17) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}λ\_{NN}-λ\_{NP}λ\_{PN}}{\sqrt{\left(λ\_{PP}+λ\_{PN}\right)\left(λ\_{NN}+λ\_{NP}\right)\left(λ\_{PP}+λ\_{NP}\frac{m\_{N}}{m\_{P}}\right)\left(λ\_{NN}+λ\_{PN}\frac{m\_{P}}{m\_{N}}\right)}}.$$ | (18) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}λ\_{NN}-\left(1-λ\_{NN}\right)\left(1-λ\_{PP}\right)}{\sqrt{1·1·\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{π\_{N}}{π\_{P}}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{π\_{P}}{π\_{N}}\right]}}.$$ | (19) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}λ\_{NN}-\left(1-λ\_{PP}-λ\_{NN}+λ\_{PP}λ\_{NN}\right)}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{π\_{N}}{π\_{P}}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{π\_{P}}{π\_{N}}\right]}}.$$ | (20) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{π\_{N}}{π\_{P}}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{π\_{P}}{π\_{N}}\right]}}.$$ | (21) |

1. Bookmaker Informedness ($BM$).

|  |  |
| --- | --- |
| $$BM≡SNS+SPC-1=λ\_{PP}+λ\_{NN}-1.$$ | (22) |

1. Markedness ($MK$).

|  |  |
| --- | --- |
| $$MK≡PPV+NPV-1=\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}+\frac{λ\_{NN}π\_{N}}{λ\_{NN}π\_{N}+\left(1-λ\_{PP}\right)π\_{P}}-1.$$ | (23) |

|  |  |
| --- | --- |
| $$MK=\frac{π\_{P}}{π\_{P}+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}π\_{N}}+\frac{π\_{N}}{π\_{N}+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}π\_{P}}-1.$$ | (24) |

## Derivation of $μ=μ\left(λ\_{PP},λ\_{NN}\right)$ when the classes are balanced.

1. Sensitivity ($SNS$).

|  |  |
| --- | --- |
| $$SNS=λ\_{PP}.$$ | (25) |

1. Specificity ($SPC$).

|  |  |
| --- | --- |
| $$SPC=λ\_{NN}.$$ | (26) |

1. Precision ($PRC$).

|  |  |
| --- | --- |
| $$PRC=\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}=\frac{λ\_{PP}·\frac{1}{2}}{λ\_{PP}·\frac{1}{2}+\left(1-λ\_{NN}\right)·\frac{1}{2}}=\frac{λ\_{PP}}{λ\_{PP}+\left(1-λ\_{NN}\right)}.$$ | (27) |

1. Negative Predictive Value ($NPV$).

|  |  |
| --- | --- |
| $$NPV=\frac{λ\_{NN}π\_{N}}{λ\_{NN}π\_{N}+\left(1-λ\_{PP}\right)π\_{P}}=\frac{λ\_{NN}·\frac{1}{2}}{λ\_{NN}·\frac{1}{2}+\left(1-λ\_{PP}\right)·\frac{1}{2}}=\frac{λ\_{NN}}{λ\_{NN}+\left(1-λ\_{PP}\right)}.$$ | (28) |

1. Accuracy ($ACC$).

|  |  |
| --- | --- |
| $$ACC=λ\_{PP}π\_{P}+λ\_{NN}π\_{N}=λ\_{PP}·\frac{1}{2}+λ\_{NN}·\frac{1}{2}=\frac{1}{2}\left(λ\_{PP}+λ\_{NN}\right).$$ | (29) |

1. $F\_{1}$ score.

|  |  |
| --- | --- |
| $$F\_{1}=2 \frac{λ\_{PP}·\frac{1}{2}}{\left(1+λ\_{PP}\right)·\frac{1}{2}+\left(1-λ\_{NN}\right)·\frac{1}{2}}= \frac{2λ\_{PP}}{\left(1+λ\_{PP}\right)+\left(1-λ\_{NN}\right)}.$$ | (30) |

1. Geometric Mean ($GM$).

|  |  |
| --- | --- |
| $$GM=\sqrt{λ\_{PP}·λ\_{NN}}.$$ | (31) |

1. Matthews Correlation Coefficient ($MCC$).

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{π\_{N}}{π\_{P}}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{π\_{P}}{π\_{N}}\right]}}.$$ | (32) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{0.5}{0.5}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{0.5}{0.5}\right]}}.$$ | (33) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\right]}}.$$ | (34) |

1. Bookmaker Informedness ($BM$).

|  |  |
| --- | --- |
| $$BM=λ\_{PP}+λ\_{NN}-1.$$ | (35) |

1. Markedness ($MK$).

|  |  |
| --- | --- |
| $$MK=\frac{π\_{P}}{π\_{P}+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}π\_{N}}+\frac{π\_{N}}{π\_{N}+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}π\_{P}}-1.$$ | (36) |

|  |  |
| --- | --- |
| $$MK=\frac{\frac{1}{2}}{\frac{1}{2}+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}·\frac{1}{2}}+\frac{\frac{1}{2}}{\frac{1}{2}+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}·\frac{1}{2}}-1.$$ | (37) |

|  |  |
| --- | --- |
| $$MK=\frac{\frac{1}{2}1}{1+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}}+\frac{1}{1+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}}-1.$$ | (38) |

## Derivation of $μ=μ\left(λ\_{PP},λ\_{NN},δ\right)$.

1. Sensitivity ($SNS$).

|  |  |
| --- | --- |
| $$SNS=λ\_{PP}.$$ | (39) |

1. Specificity ($SPC$).

|  |  |
| --- | --- |
| $$SPC=λ\_{NN}.$$ | (40) |

1. Precision ($PRC$).

|  |  |
| --- | --- |
| $$PRC=\frac{λ\_{PP}π\_{P}}{λ\_{PP}π\_{P}+\left(1-λ\_{NN}\right)π\_{N}}=\frac{λ\_{PP}\frac{1+δ}{2}}{λ\_{PP}\frac{1+δ}{2}+\left(1-λ\_{NN}\right)\frac{1-δ}{2}}.$$ | (41) |

|  |  |
| --- | --- |
| $$PRC=\frac{λ\_{PP}\left(1+δ\right)}{λ\_{PP}\left(1+δ\right)+\left(1-λ\_{NN}\right)\left(1-δ\right)}.$$ | (42) |

1. Negative Predictive Value ($NPV$).

|  |  |
| --- | --- |
| $$NPV=\frac{λ\_{NN}π\_{N}}{λ\_{NN}π\_{N}+\left(1-λ\_{PP}\right)π\_{P}}=\frac{λ\_{NN}\frac{1-δ}{2}}{λ\_{NN}\frac{1-δ}{2}+\left(1-λ\_{PP}\right)\frac{1+δ}{2}}.$$ | (43) |

|  |  |
| --- | --- |
| $$NPV=\frac{λ\_{NN}\left(1-δ\right)}{λ\_{NN}\left(1-δ\right)+\left(1-λ\_{PP}\right)\left(1+δ\right)}.$$ | (44) |

1. Accuracy ($ACC$).

|  |  |
| --- | --- |
| $$ACC=λ\_{PP}\frac{1+δ}{2}+λ\_{NN}\frac{1-δ}{2}=\frac{1}{2}\left[λ\_{PP}\left(1+δ\right)+λ\_{NN}\left(1-δ\right)\right].$$ | (45) |

1. $F\_{1}$ score.

|  |  |
| --- | --- |
| $$F\_{1}=2 \frac{λ\_{PP}\frac{1+δ}{2}}{\left(1+λ\_{PP}\right)\frac{1+δ}{2}+\left(1-λ\_{NN}\right)\frac{1-δ}{2}}=\frac{2 λ\_{PP}\left(1+δ\right)}{\left(1+λ\_{PP}\right)\left(1+δ\right)+\left(1-λ\_{NN}\right)\left(1-δ\right)}.$$ | (46) |

1. Geometric Mean ($GM$).

|  |  |
| --- | --- |
| $$GM≡\sqrt{SNS·SPC}=\sqrt{λ\_{PP}·λ\_{NN}}.$$ | (47) |

1. Matthews Correlation Coefficient ($MCC$).

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{π\_{N}}{π\_{P}}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{π\_{P}}{π\_{N}}\right]}}.$$ | (48) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{\frac{1-δ}{2}}{\frac{1+δ}{2}}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{\frac{1+δ}{2}}{\frac{1-δ}{2}}\right]}}.$$ | (49) |

|  |  |
| --- | --- |
| $$MCC=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{1-δ}{1+δ}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{1+δ}{1-δ}\right]}}.$$ | (50) |

1. Bookmaker Informedness ($BM$).

|  |  |
| --- | --- |
| $$BM=λ\_{PP}+λ\_{NN}-1.$$ | (51) |

1. Markedness ($MK$).

|  |  |
| --- | --- |
| $$MK=MK=\frac{π\_{P}}{π\_{P}+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}π\_{N}}+\frac{π\_{N}}{π\_{N}+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}π\_{P}}-1.$$ | (52) |

|  |  |
| --- | --- |
| $$MK=MK=\frac{\frac{1+δ}{2}}{\frac{1+δ}{2}+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}\frac{1-δ}{2}}+\frac{\frac{1-δ}{2}}{\frac{1-δ}{2}+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}\frac{1+δ}{2}}-1.$$ | (53) |

|  |  |
| --- | --- |
| $$MK=MK=\frac{1+δ}{\left(1+δ\right)+\frac{\left(1-λ\_{NN}\right)}{λ\_{PP}}\left(1-δ\right)}+\frac{1-δ}{\left(1-δ\right)+\frac{\left(1-λ\_{PP}\right)}{λ\_{NN}}\left(1+δ\right)}-1.$$ | (54) |

## Derivation of $B\_{μ}=B\_{μ}\left(λ\_{PP},λ\_{NN},δ\right)$.

1. Sensitivity ($SNS$).

|  |  |
| --- | --- |
| $$B\_{SNS}\left(λ\_{PP},λ\_{NN},δ\right)=SNS\left(λ\_{PP},λ\_{NN},δ\right)-SNS\_{b}\left(λ\_{PP},λ\_{NN},δ\right)=λ\_{PP}-λ\_{PP}=0.$$ | (55) |

1. Specificity ($SPC$).

|  |  |
| --- | --- |
| $$B\_{SPC}\left(λ\_{PP},λ\_{NN},δ\right)=SPC\left(λ\_{PP},λ\_{NN},δ\right)-SPC\_{b}\left(λ\_{PP},λ\_{NN},δ\right)=λ\_{NN}-λ\_{NN}=0.$$ | (56) |

1. Precision ($PRC$).

|  |  |
| --- | --- |
| $$B\_{PRC}\left(λ\_{PP},λ\_{NN},δ\right)=PRC\left(λ\_{PP},λ\_{NN},δ\right)-PRC\_{b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (57) |

|  |  |
| --- | --- |
| $$B\_{PRC}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{λ\_{PP}\left(1+δ\right)}{λ\_{PP}\left(1+δ\right)+\left(1-λ\_{NN}\right)\left(1-δ\right)}-\frac{λ\_{PP}}{λ\_{PP}+\left(1-λ\_{NN}\right)}.$$ | (58) |

|  |  |
| --- | --- |
| $$B\_{PRC}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{1+δ}{\left(1+δ\right)+\frac{1-λ\_{NN}}{λ\_{PP}}\left(1-δ\right)}-\frac{1}{1+\frac{1-λ\_{NN}}{λ\_{PP}}}.$$ | (59) |

1. Negative Predictive Value ($NPV$).

|  |  |
| --- | --- |
| $$B\_{NPV}\left(λ\_{PP},λ\_{NN},δ\right)=NPV\left(λ\_{PP},λ\_{NN},δ\right)-NPV\_{b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (60) |

|  |  |
| --- | --- |
| $$B\_{NPV}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{λ\_{NN}\left(1-δ\right)}{λ\_{NN}\left(1-δ\right)+\left(1-λ\_{PP}\right)\left(1+δ\right)}-\frac{λ\_{NN}}{λ\_{NN}+\left(1-λ\_{PP}\right)}.$$ | (61) |

|  |  |
| --- | --- |
| $$B\_{NPV}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{1-δ}{\left(1-δ\right)+\frac{1-λ\_{PP}}{λ\_{NN}}\left(1+δ\right)}-\frac{1}{1+\frac{1-λ\_{PP}}{λ\_{NN}}}.$$ | (62) |

1. Accuracy ($ACC$).

|  |  |
| --- | --- |
| $$B\_{ACC}\left(λ\_{PP},λ\_{NN},δ\right)=ACC\left(λ\_{PP},λ\_{NN},δ\right)-ACC\_{b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (63) |

|  |  |
| --- | --- |
| $$B\_{ACC}\left(λ\_{PP},λ\_{NN},δ\right)=λ\_{PP}\frac{1+δ}{2}+λ\_{NN}\frac{1-δ}{2}-\frac{λ\_{PP}+λ\_{NN}}{2}.$$ | (64) |

|  |  |
| --- | --- |
| $$B\_{ACC}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{λ\_{PP}}{2}+\frac{λ\_{PP}δ}{2}+\frac{λ\_{NN}}{2}-\frac{λ\_{NN}δ}{2}-\frac{λ\_{PP}}{2}-\frac{λ\_{NN}}{2}=\frac{λ\_{PP}δ}{2}-\frac{λ\_{NN}δ}{2}.$$ | (65) |

|  |  |
| --- | --- |
| $$B\_{ACC}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{δ}{2}\left(λ\_{PP}-λ\_{NN}\right).$$ | (66) |

1. $F\_{1}$ score.

|  |  |
| --- | --- |
| $$B\_{F1}\left(λ\_{PP},λ\_{NN},δ\right)=F\_{1}\left(λ\_{PP},λ\_{NN},δ\right)-F\_{1b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (67) |

|  |  |
| --- | --- |
| $$B\_{F1}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{2λ\_{PP}\left(1+δ\right)}{\left(1+λ\_{PP}\right)\left(1+δ\right)+\left(1-λ\_{NN}\right)\left(1-δ\right)}-\frac{2λ\_{PP}}{2+λ\_{PP}-λ\_{NN}}.$$ | (68) |

1. Geometric Mean ($GM$).

|  |  |
| --- | --- |
| $$B\_{GM}\left(λ\_{PP},λ\_{NN},δ\right)=GM\left(λ\_{PP},λ\_{NN},δ\right)-GM\_{b}\left(λ\_{PP},λ\_{NN},δ\right)=\sqrt{λ\_{PP}·λ\_{NN}}-\sqrt{λ\_{PP}·λ\_{NN}}=0.$$ | (69) |

1. Matthews Correlation Coefficient ($MCC$).

|  |  |
| --- | --- |
| $$B\_{MCC}\left(λ\_{PP},λ\_{NN},δ\right)=MCC\left(λ\_{PP},λ\_{NN},δ\right)-MCC\_{b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (70) |

|  |  |
| --- | --- |
| $$B\_{MCC}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{1-δ}{1+δ}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{1+δ}{1-δ}\right]}}-\frac{λ\_{PP}+λ\_{NN}-1}{\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\right]}}.$$ | (71) |

|  |  |
| --- | --- |
| $$B\_{MCCn}\left(λ\_{PP},λ\_{NN},δ\right)=MCCn\left(λ\_{PP},λ\_{NN},δ\right)-MCCn\_{b}\left(λ\_{PP},λ\_{NN}\right).$$ | (72) |

|  |  |
| --- | --- |
| $$B\_{MCCn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{MCC\left(λ\_{PP},λ\_{NN},δ\right)+1}{2}-\frac{MCC\_{b}\left(λ\_{PP},λ\_{NN}\right)+1}{2}.$$ | (73) |

|  |  |
| --- | --- |
| $$B\_{MCCn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{MCC\left(λ\_{PP},λ\_{NN},δ\right)-MCC\_{b}\left(λ\_{PP},λ\_{NN}\right)}{2}.$$ | (74) |

|  |  |
| --- | --- |
| $$B\_{MCCn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{B\_{MCC}\left(λ\_{PP},λ\_{NN},δ\right)}{2}.$$ | (75) |

|  |  |
| --- | --- |
| $$B\_{MCCn}\left(λ\_{PP},λ\_{NN},δ\right)==\frac{λ\_{PP}+λ\_{NN}-1}{2\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\frac{1-δ}{1+δ}\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\frac{1+δ}{1-δ}\right]}}-\frac{λ\_{PP}+λ\_{NN}-1}{2\sqrt{\left[λ\_{PP}+\left(1-λ\_{NN}\right)\right]\left[λ\_{NN}+\left(1-λ\_{PP}\right)\right]}}.$$ | (76) |

1. Bookmaker Informedness ($BM$).

|  |  |
| --- | --- |
| $$B\_{BM}\left(λ\_{PP},λ\_{NN},δ\right)=BM\left(λ\_{PP},λ\_{NN},δ\right)-BM\_{b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (77) |

|  |  |
| --- | --- |
| $$B\_{BM}\left(λ\_{PP},λ\_{NN},δ\right)=\left(λ\_{PP}+λ\_{NN}-1\right)-\left(λ\_{PP}+λ\_{NN}-1\right)=0.$$ | (78) |

1. Markedness ($MK$).

|  |  |
| --- | --- |
| $$B\_{MK}\left(λ\_{PP},λ\_{NN},δ\right)=MK\left(λ\_{PP},λ\_{NN},δ\right)-MK\_{b}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (79) |

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| $$B\_{MK}\left(λ\_{PP},λ\_{NN},δ\right)=\left[PRC\left(λ\_{PP},λ\_{NN},δ\right)+NPV\left(λ\_{PP},λ\_{NN},δ\right)-1\right]-\left[PRC\_{b}\left(λ\_{PP},λ\_{NN}\right)+NPV\_{b}\left(λ\_{PP},λ\_{NN}\right)-1\right].$$ | (80) |

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| $$B\_{MK}\left(λ\_{PP},λ\_{NN},δ\right)=\left[PRC\left(λ\_{PP},λ\_{NN},δ\right)-PRC\_{b}\left(λ\_{PP},λ\_{NN}\right)\right]+\left[NPV\left(λ\_{PP},λ\_{NN},δ\right)-NPV\_{b}\left(λ\_{PP},λ\_{NN}\right)\right].$$ | (81) |

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| $$B\_{MK}\left(λ\_{PP},λ\_{NN},δ\right)=B\_{PRC}\left(λ\_{PP},λ\_{NN},δ\right)+B\_{NPV}\left(λ\_{PP},λ\_{NN},δ\right).$$ | (82) |

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| $$B\_{MK}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{1+δ}{\left(1+δ\right)+\frac{1-λ\_{NN}}{λ\_{PP}}\left(1-δ\right)}-\frac{1}{1+\frac{1-λ\_{NN}}{λ\_{PP}}}+\frac{1-δ}{\left(1-δ\right)+\frac{1-λ\_{PP}}{λ\_{NN}}\left(1+δ\right)}-\frac{1}{1+\frac{1-λ\_{PP}}{λ\_{NN}}}.$$ | (83) |

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| $$B\_{MKn}\left(λ\_{PP},λ\_{NN},δ\right)=MKn\left(λ\_{PP},λ\_{NN},δ\right)-MKn\_{b}\left(λ\_{PP},λ\_{NN}\right).$$ | (84) |

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| --- | --- |
| $$B\_{MKn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{MK\left(λ\_{PP},λ\_{NN},δ\right)+1}{2}-\frac{MK\_{b}\left(λ\_{PP},λ\_{NN}\right)+1}{2}.$$ | (85) |

|  |  |
| --- | --- |
| $$B\_{MKn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{MK\left(λ\_{PP},λ\_{NN},δ\right)-MK\_{b}\left(λ\_{PP},λ\_{NN}\right)}{2}=\frac{B\_{MK}\left(λ\_{PP},λ\_{NN},δ\right)}{2}.$$ | (86) |

|  |  |
| --- | --- |
| $$B\_{MKn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{B\_{PRC}\left(λ\_{PP},λ\_{NN},δ\right)+B\_{NPV}\left(λ\_{PP},λ\_{NN},δ\right)}{2}.$$ | (87) |

|  |  |
| --- | --- |
| $$B\_{MKn}\left(λ\_{PP},λ\_{NN},δ\right)=\frac{1}{2}\left(\frac{1+δ}{\left(1+δ\right)+\frac{1-λ\_{NN}}{λ\_{PP}}\left(1-δ\right)}-\frac{1}{1+\frac{1-λ\_{NN}}{λ\_{PP}}}+\frac{1-δ}{\left(1-δ\right)+\frac{1-λ\_{PP}}{λ\_{NN}}\left(1+δ\right)}-\frac{1}{1+\frac{1-λ\_{PP}}{λ\_{NN}}}\right).$$ | (88) |